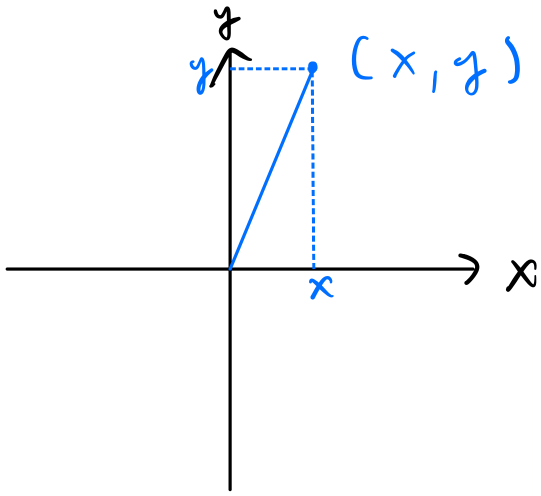
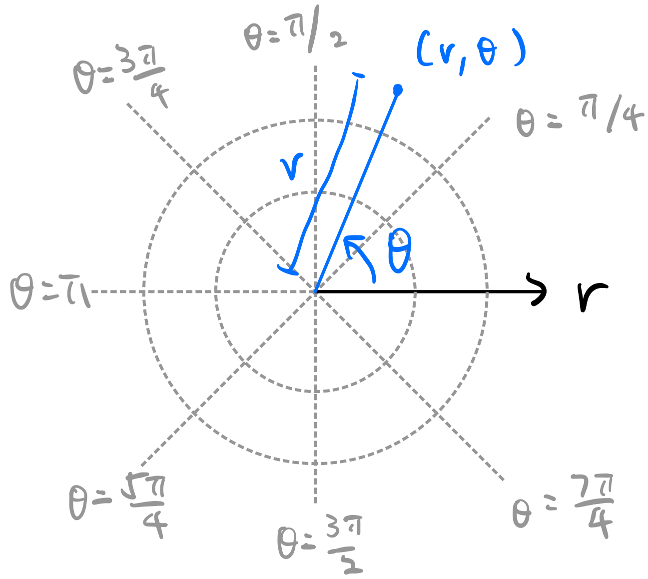
Polar Coordinates ($\mathbb{R}^2$)

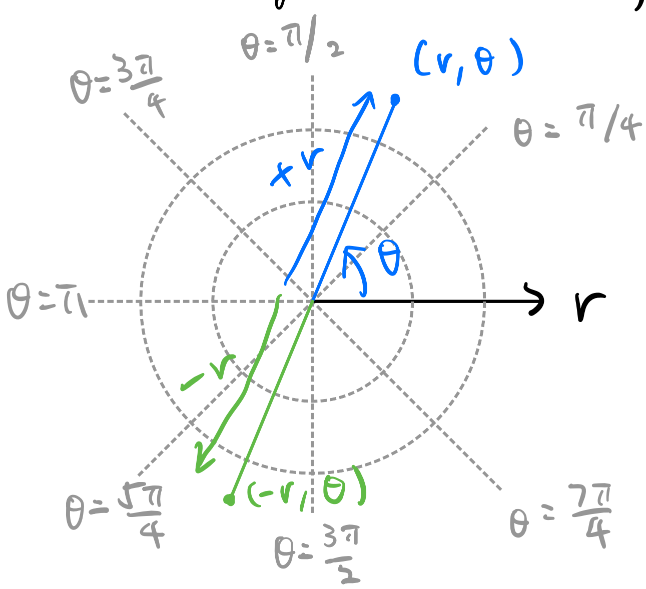
Cartesian / Rectangular :

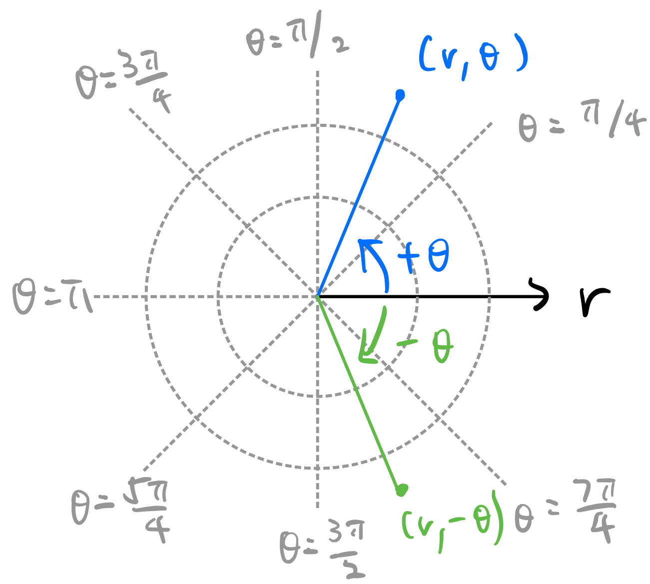


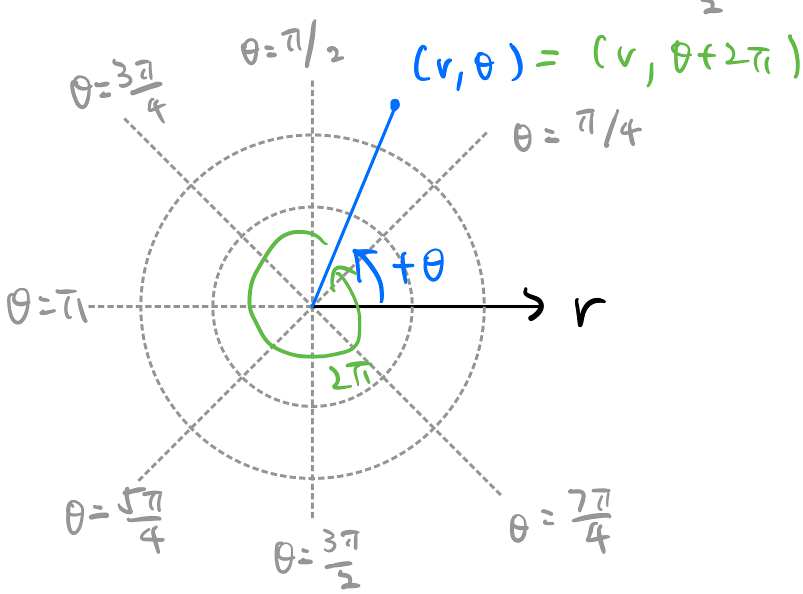
Polar Coordinate.



$$x = r\cos\theta$$
$$y = r\sin\theta \quad , \quad -\infty < r, \theta < \infty$$







§ 11.3

68. Vertical and horizontal lines

a. Show that every vertical line in the xy-plane has a polar equation of the form $r = a \sec \theta$.

b. Find the analogous polar equation for horizontal lines in the xy-plane.

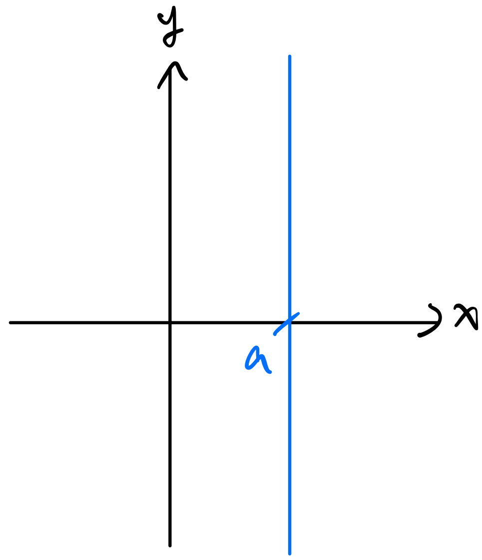
(a). vertical line :

$x = a$, for some $a \in \mathbb{R}$.

also , $x = r\cos\theta$

$\therefore$ $a = r\cos\theta$

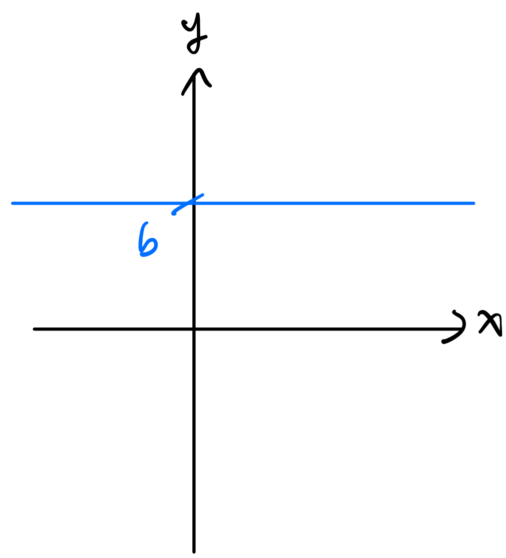
$\Rightarrow$ $r = a\sec\theta$.



(b). Let $y = b$ be a horizontal line.

$b = y = r\sin\theta$

$\Rightarrow$ $r = b\csc\theta$.



§ 11.4

Find the slopes of the curves in Exercises 17–20 at the given points. Sketch the curves along with their tangents at these points.

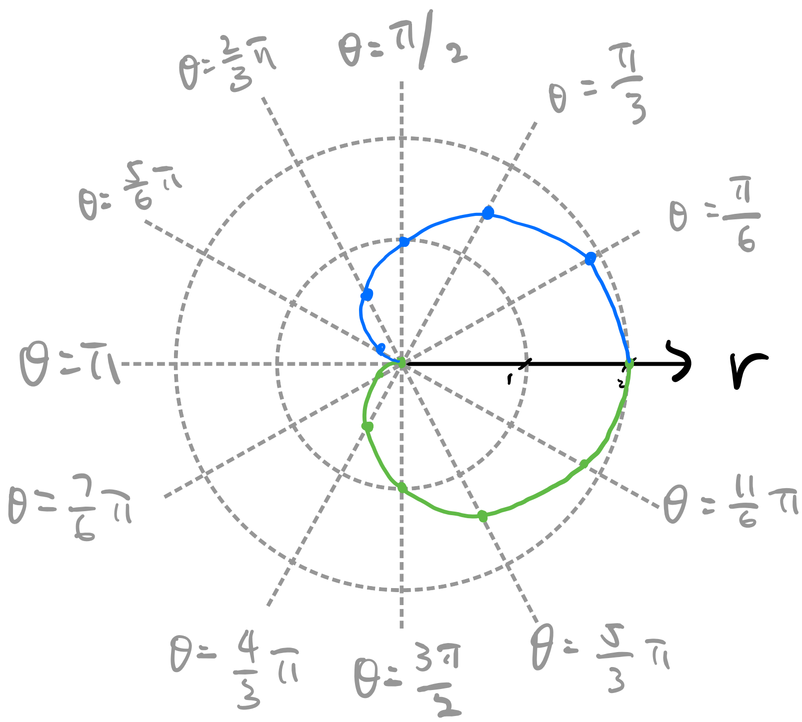
17. Cardioid $r = -1 + \cos\theta;\quad \theta = \pm\pi/2$

Sketching :

For $0 \le \theta \le \pi$:

θ	r
0	0
$\pi/6$	≈ -0.1
$\pi/3$	-0.5
$\pi/2$	-1
$2\pi/3$	-1.5
$5\pi/6$	≈ -1.9
π	-2

$\Rightarrow$ Green Curve



$r_\theta = -1 + \cos\theta$

$= -1 + \cos(-\theta)$

$= r_{-\theta}$

$\Rightarrow$ Symmetric about x-axis / $\theta = 0$

$\Rightarrow$ Blue Curve

$r_\theta = -1 + \cos\theta$

$= -1 + \cos(\theta + 2\pi)$

$= r_{\theta + 2\pi}$

$\Rightarrow$ 2π periodic.

$$\text{slope} = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

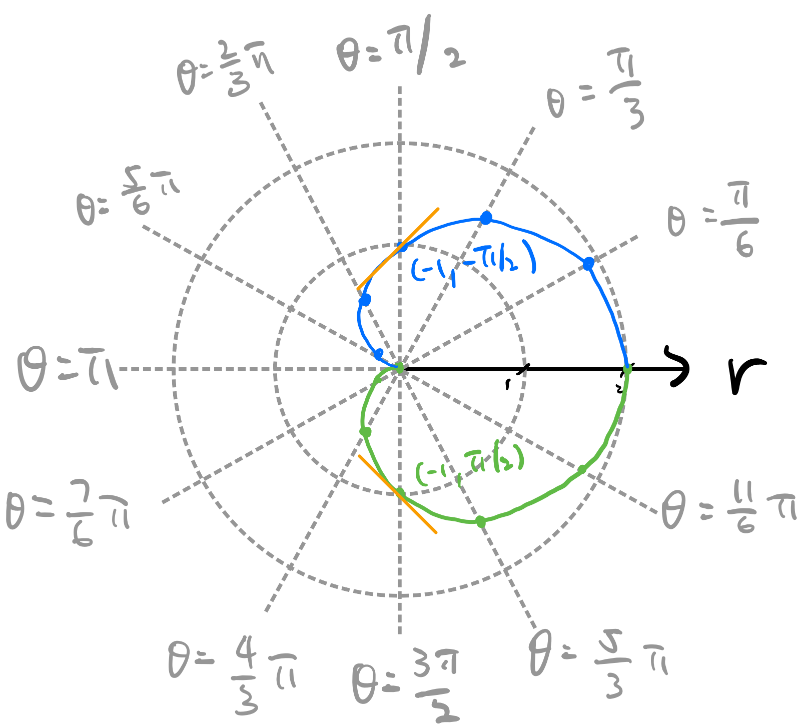
$$= \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)}$$

$$= \frac{\frac{d}{d\theta}\left((-1+\cos\theta)\sin\theta\right)}{\frac{d}{d\theta}\left((-1+\cos\theta)\cos\theta\right)}$$

$$= \frac{r(\cos\theta) + (-\sin\theta)\sin\theta}{r(-\sin\theta) + (-\sin\theta)\cos\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = -1 \qquad \left.\frac{dy}{dx}\right|_{\theta=-\frac{\pi}{2}} = 1$$

Tangents:



Graph the limaçons in Exercises 21–24. Limaçon ("*lee*-ma-sahn") is Old French for "snail." You will understand the name when you graph the limaçons in Exercise 21. Equations for limaçons have the form $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$. There are four basic shapes.

21. Limaçons with an inner loop

a. $r = \frac{1}{2} + \cos\theta$ **b.** $r = \frac{1}{2} + \sin\theta$

For $0 \leq \theta \leq \pi$:

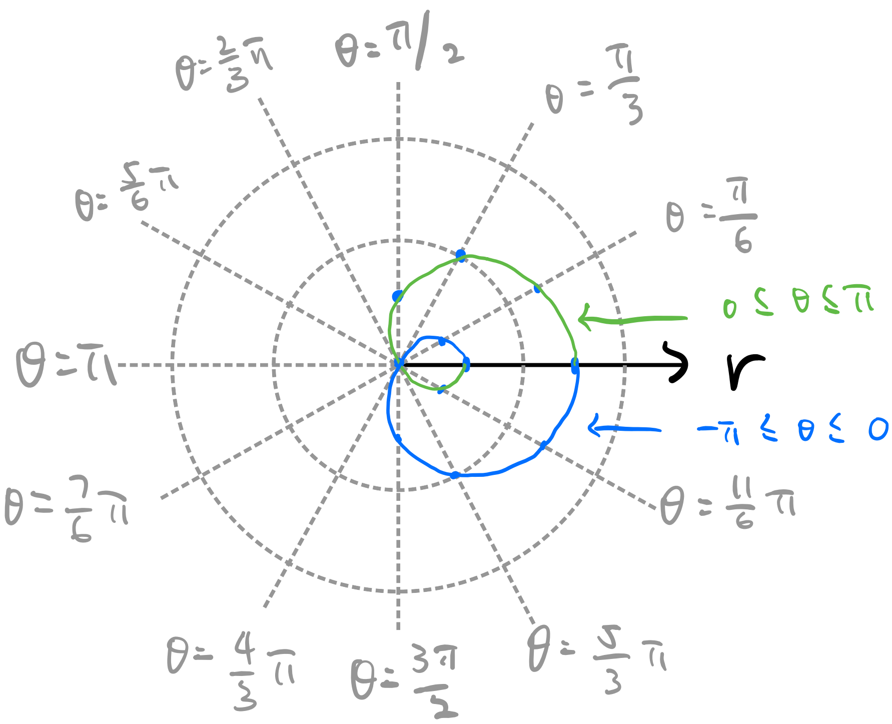
a:

θ	r
0	1.5
$\pi/6$	≈ 1.4
$\pi/3$	1
$\pi/2$	0.5
$2\pi/3$	0
$5\pi/6$	-0.4
π	-0.5

$$r^a_\theta = \frac{1}{2} + \cos\theta = \frac{1}{2} + \cos(-\theta) = r^a_{-\theta}$$

$\Rightarrow$ Curve (a) is symmetric about x-axis / $\theta = 0$

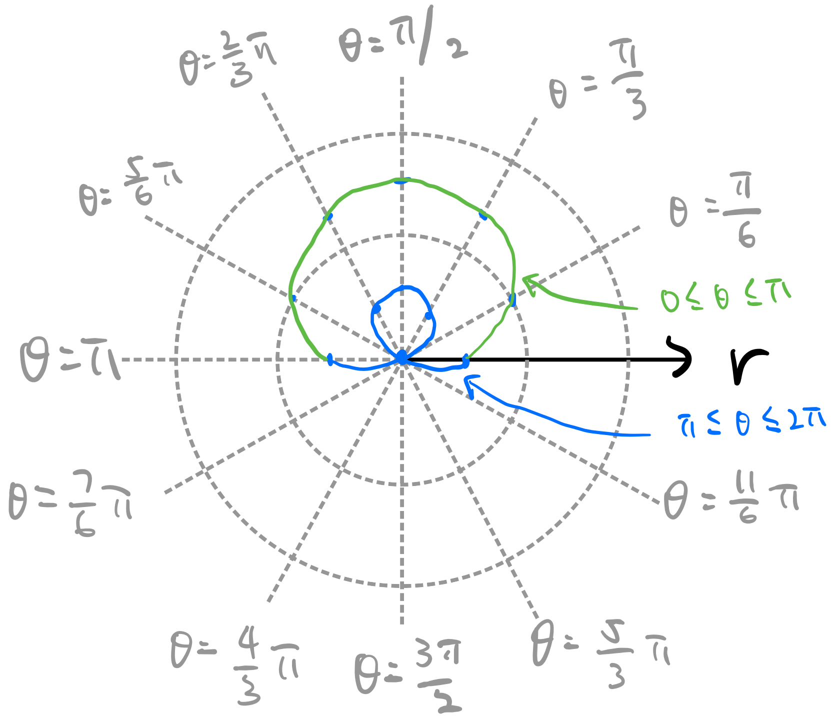
Also, Curve (a) is 2π-periodic.



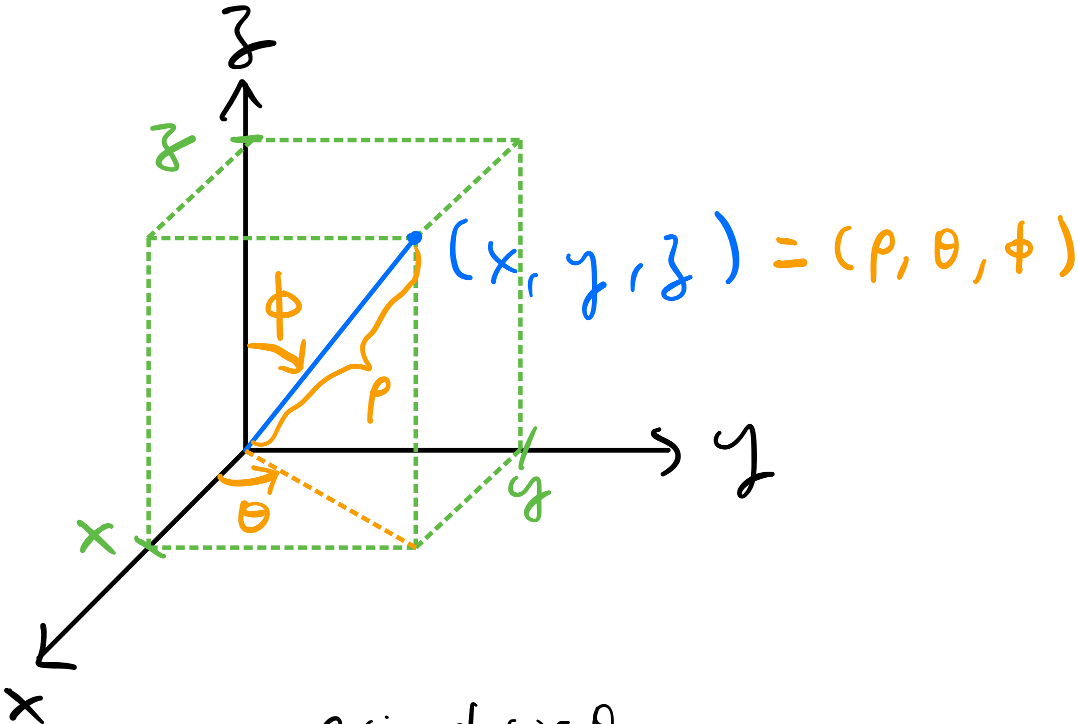
b:

θ	r
0	0.5
$\pi/6$	1
$\pi/3$	≈ 1.4
$\pi/2$	1.5
$2\pi/3$	≈ 1.4
$5\pi/6$	1
π	0.5

θ	r
$7\pi/6$	0
$4\pi/3$	≈ -0.4
$3\pi/2$	-0.5
$5\pi/3$	≈ -0.4
$11\pi/6$	0

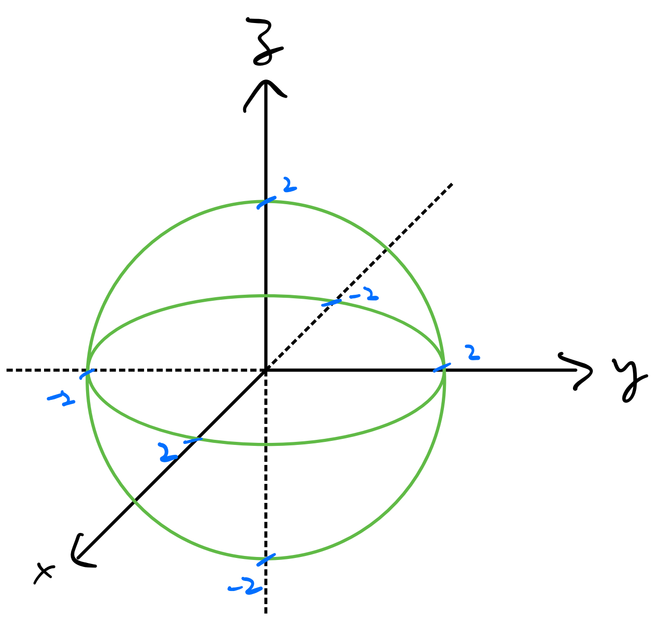


Spherical Coordinates ($\mathbb{R}^3$).

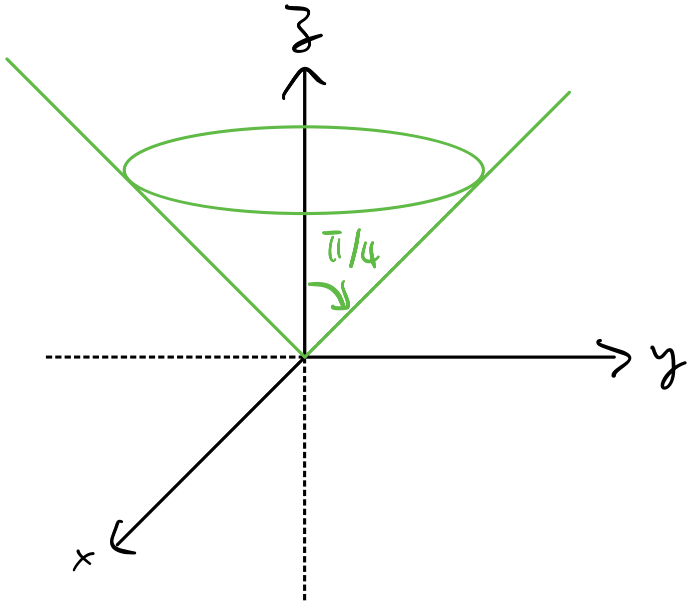


$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\theta$$

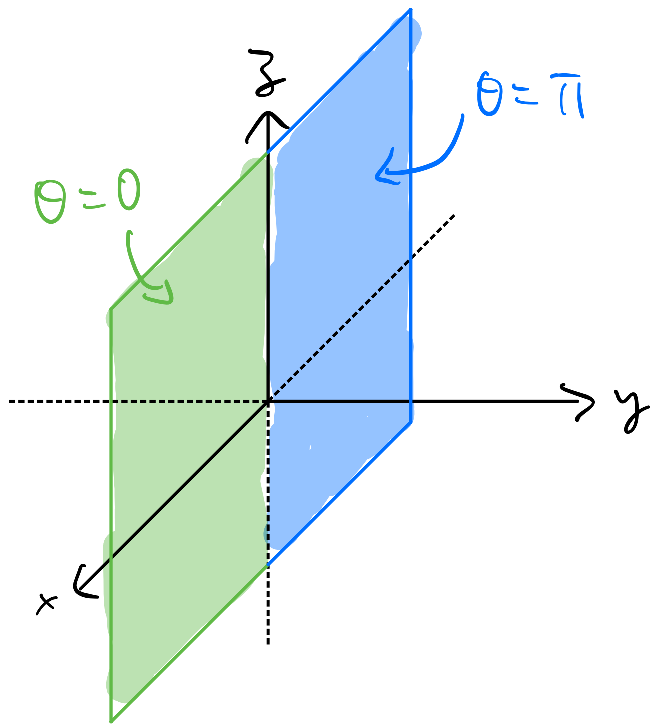
e.g. (1) $\rho = 2$: sphere with radius 2, centred at origin.



(2) : $\phi = \frac{\pi}{4}$ Cone

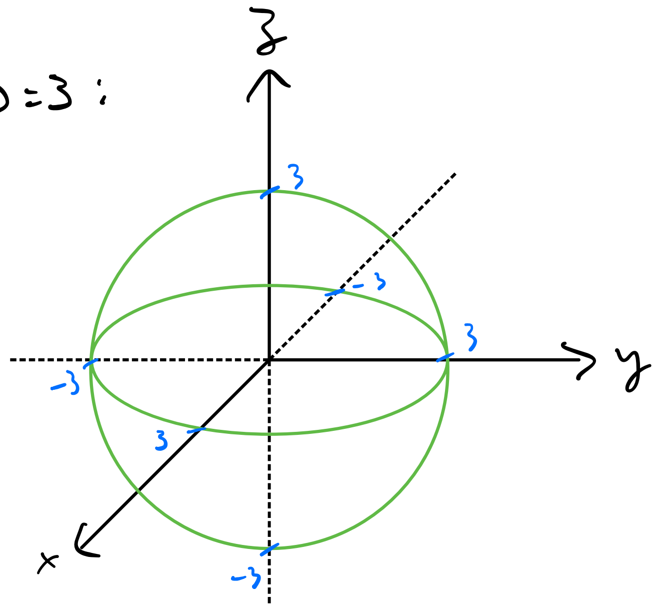


(3) : $\theta = 0$: half plane : $x > 0,\ y = 0,\ z \in \mathbb{R}$



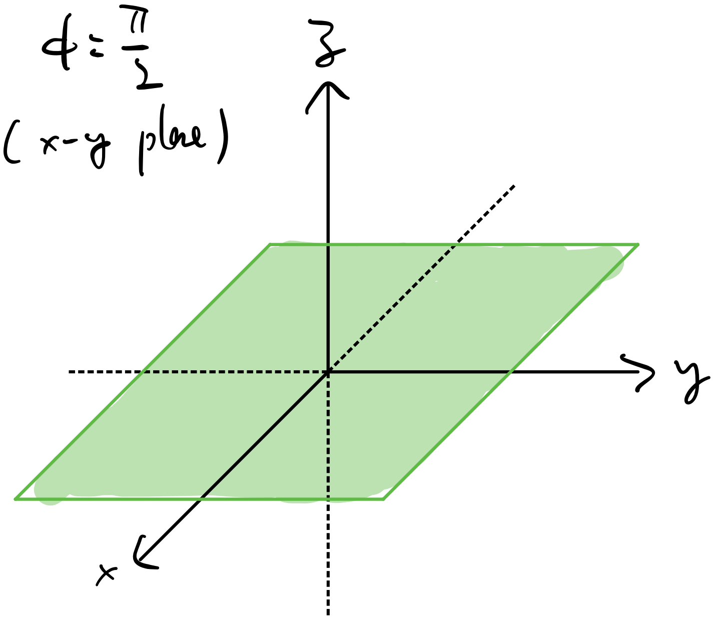
(4) $\rho = 3$, $\phi = \frac{\pi}{2}$

$\rho = 3$:



$\phi = \frac{\pi}{2}$

(x-y plane)



Intersection

$\Downarrow$